Indian Statistical Institute, Bangalore

Mathematics of Computing

Mid Term Examination, March 2014 Max Marks:20; Weightage 20%, Max Time: 2 hrs

1) Consider L_0 as the language consisting of strings on the alphabet $\{a,b\}$ which are at least two long and have an a in the second last position: *e.g.,* the language contains "*bbab*" but neither "*aaba*" nor "*a*". Here is a regular expression r for the strings in L_0 : $r=(a|b)^*a(a|b)$.
   a) Create an NFA from r for L_0. (1)
   b) Convert your NFA to a DFA (2)
   c) Give a right linear grammar for L_0. (1)
2) If L_a *and* L_b are regular then show that XOR(L_a, L_b), which is the language that contains strings in exactly one of the two languages, is regular.(1)
3) Show how to convert a given an NFA M_a for a regular language L to an NPDA M_p for L. (2)
4) Prove that one can construct a NPDA for an arbitrary CFL. Assume the CFL does not contain the empty string.(3)
5) Consider the language $L_1=\{a^n b^m c^n : n,m \geq 1\}$:
   a) Is L_1 Regular? If it is, construct a right linear grammar for L_1, if not use the pumping lemma to prove your claim. (3)
   b) Is L_1 Context-Free? If it is, construct a CFG for L_1, if not use the pumping lemma to prove your claim. (3)
6) Show the following language is not context free by using the pumping lemma for context free languages: $L_r=\{ww : w \text{ is a string on } \{a,b\}^*\}$. (4)